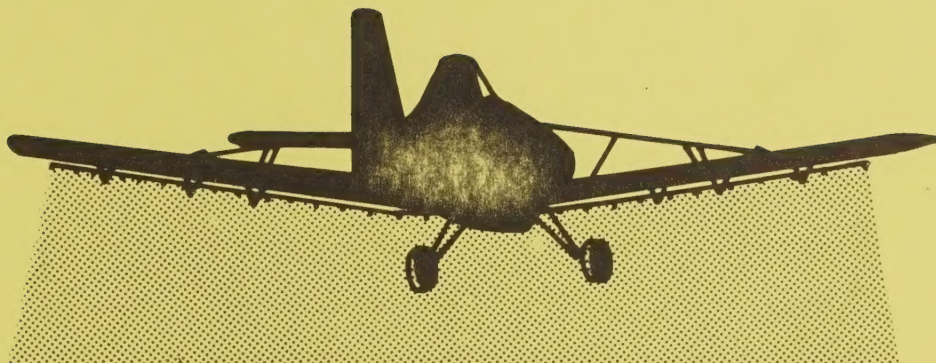


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A Summary of Analytic Expressions for Droplet Evaporation

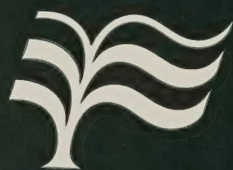
ED&T 2664
Optimum Size Spray Droplets

November 1979



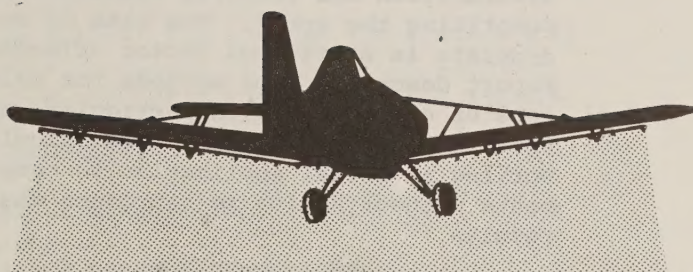
Forest Service
U.S. Department of Agriculture
Equipment Development Center
Fort Missoula
Missoula, Montana 59801

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A Summary of Analytic Expressions for Droplet Evaporation

by
Robert Banaugh
Mechanical Engineer

ABSTRACT

Predicting the effectiveness of an aerially sprayed pesticide requires a method for calculating pesticide concentration and the distribution and temporal variation of the size of the droplets comprising the spray. The rate of evaporation of the spray droplets is a critical factor affecting these calculations. This report describes some methods for calculating the evaporation rate of a droplet, with particular emphasis on how the governing equations were derived. These calculations will enable those working in aerial application to compare data from other experiments without having to reproduce the tests to evaluate the results.

A report on Equipment Development and Test Project 2664, Optimum Size Spray Droplets, funded by the Forest Insect and Disease Management Staff.

PESTICIDE PRECAUTIONARY STATEMENT

This publication reports research involving pesticides. It does not contain recommendations for their use, nor does it imply that the uses discussed here have been registered. All uses of pesticides must be registered by appropriate State and/or Federal agencies before they can be recommended.

CAUTION: Pesticides can be injurious to humans, domestic animals, desirable plants, and fish or other wildlife — if they are not handled or applied properly. Use all pesticides selectively and carefully. Follow recommended practices for the disposal of surplus pesticides and pesticide containers.



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INTRODUCTION

An important objective of the Forest Service aerial applications program is the development of a technique for predicting the behavior of a pesticide spray once it has been released from the aircraft. This prediction requires a description of the temporal and spatial distribution of the droplets comprising the spray. A critical factor affecting this prediction is the evaporation rate of the spray droplets. Because droplet evaporation data are exceedingly difficult to obtain, the Missoula Equipment Development Center (MEDC) has contracted with an outside agency to collect data from evaporation experiments. To assess the validity of these data, Center engineers investigated several theoretical models, summarized in this report, for predicting droplet evaporation rates.

No satisfactory theoretical model yet exists for predicting the evaporation rates of multiphase liquid droplets or droplets containing particulate matter. Experiments with these types of droplets have yielded only scattered results. Consequently, calculations for evaporation rates in this report have been based on pure liquid droplets so that the results can serve as a standard for assessing evaporation rate data obtained from actual experiments.

Two theoretical models for predicting the evaporation rates of pure liquid droplets have been found to produce results that are in close agreement with results obtained from droplet evaporation experiments. These models, based on the works of J.C. Maxwell, I. Langmuir, and N.A. Fuchs, are summarized in this report. For each model, the assumptions used to derive the governing equations have been carefully stated and the range of applicability specified.

In addition to excluding multiphase liquid droplets and particulate droplets from the theoretical calculations for droplet evaporation rates, the effects of condensation and the influence of gaseous droplets adjacent to the spray cloud also have been excluded.

Evaporation is the process where molecules of a liquid leave the liquid and join the surrounding gaseous or vacuous state. It also may be the case that some of the vapor molecules of the surrounding medium collide with the droplet surface and thus join the liquid. This process is termed condensation. In this report, it will be assumed that the process of condensation may be ignored, however, this assumption may not be valid for all aerial spray applications. For example, if the spray carrier were water-based and the relative humidity of the air were high, a thorough discussion of droplet evaporation would have to include the effects of condensation. Condensation also may be important in spray clouds that are very dense because evaporation from one set of cloud particles will contribute to condensation on another set of cloud droplets. Nevertheless, in this report, emphasis will be given to examining expressions for the evaporation rate of single droplets and condensation effects will not be considered.

From the laws of hydrodynamics and thermodynamics it is known that pressure and temperature gradients in both the surrounding gas and the droplet govern the motion of the molecules comprising the gas and the liquid droplets. A discussion of the mass transfer processes induced by these gradients must be derived from a statistical mechanical description of the behavior of matter. This description has not yet been completely worked out. Consequently, this report will adopt the macroscopic or continuum mechanics approach rather than the particle or statistical mechanics approach.

Droplet evaporation is only one of many factors that affects the behavior of pesticide sprays. Other phenomena, such as topography, wind velocity, and turbulence are equally important. Thus, the theoretical models in this report provide a satisfactory "first approximation" for accounting for the affect of droplet evaporation on the behavior of aerial sprays.

The first portion of this work will present the development of expressions

for determining the evaporation rates of droplets that are at rest with respect to the surrounding medium. In the second portion of the report, expressions for the evaporation rates of droplets that are moving with respect to the surrounding medium will then be obtained by modifying the previously developed "at rest" results.

An excellent survey and critique of experimental methods for determining evaporation rates has been given by Beard and Pruppacher (1971). Their paper also presents useful equations for pure water droplets. Pruppacher and Rasmussen (1979) have recently extended the previous work to include the determination of the rate of evaporation of large water drops falling at terminal velocity in air. Woo and Hamielec (1971) present an accurate numerical method determining evaporation rates and discuss the work of other researchers.

EVAPORATION OF STATIONARY DROPLETS

Maxwell-Langmuir Model of Evaporation

The earliest and simplest model of evaporation is Maxwell's (1890). His results were used by Langmuir (1918) to obtain an equation governing the time rate of change of the mass of an evaporating droplet at rest with respect to the surrounding medium. The Langmuir equation may be used to derive an equation for the temporal variation of the droplet mass, surface area, and radius. To enable the reader to more easily follow the derivation of the model equations, we first give a brief outline of the development.

1. The evaporating droplet is assumed to produce an infinitely thin shell of vapor molecules which then move or diffuse out into the surrounding medium. The concentration of molecules in the shell is assumed to be at the saturation point.

2. From assumptions governing the medium and the shape of the droplet during the evaporation process, an expression is derived for the rate at which mass is transferred from the shell to the surrounding medium.

3. This expression is equated to the time rate of change of the droplet mass to give Langmuir's equation.

4. Langmuir's equation is then integrated to give the temporal variation of the droplet surface area, mass, and radius.

Development of the Equations

Assumptions:

- The droplet is spherical and remains so during the entire evaporation process.
- The droplet is motionless with respect to the infinite medium assumed to surround the droplet.
- The process of evaporation results in a concentration or density of evaporating molecules in the spherical shell immediately surrounding the droplet and this concentration is equal to the vapor concentration of a saturated vapor corresponding to the temperature of the droplet.
- Diffusion is the mechanism governing the transport of the molecules in the spherical shell to the surrounding medium.

Derivation:

The mixing of two substances by the passage of molecules from one substance through spaces between the molecules of a second substance is called diffusion. Diffusion is a mass transfer process since a mass of molecules leaves one substance to intermingle or to be transferred to the space occupied by the molecules of a second substance. This suggests that an important variable in describing a diffusion process is the concentration of molecules, that is, the number of molecules per unit volume. It will be postulated that molecules move from a region of higher concentration to a region of lower concentration at a rate which is proportional to the difference in concentrations. Let C denote the concentration of vapor molecules and let \dot{J} denote the mass of vapor molecules passing through a unit area normal to the vapor mass flux in unit time. \dot{J} is expressed in $\text{gm cm}^{-2} \text{sec}^{-1}$.

Now, for a spherical surface of radius r (cm)

$$\vec{J} \cdot \vec{ds}$$

is the mass flux (gm sec^{-1}) leaving the surface through an element of surface area, ds , in a direction perpendicular to ds . The symbol ds denotes a vector whose magnitude is the area (cm^2) of the small increment of the surface through which the particles are passing and whose direction is perpendicular to the small area increment. Because the surface is assumed to be spherical and to retain its spherical shape throughout the evaporation process, if spherical coordinates r , θ , and ϕ are introduced, the magnitude of an element of surface area is given by

$$r^2 \sin\theta \, d\theta \, d\phi$$

The mass flux of vapor molecules through this area is then

$$\vec{J} \cdot \hat{n} r^2 \sin\theta \, d\theta \, d\phi$$

where \hat{n} is the unit direction vector normal to the surface element, ds . Hence, the mass flux, F , of vapor through the entire spherical surface is

$$F = \int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{\phi=0}^{\phi=2\pi} \vec{J} \cdot \hat{n} r^2 \sin\theta \, d\theta \, d\phi \quad (1)$$

Because the vapor flux of molecules is assumed to be directly proportional to the gradient of the vapor concentration, C , it follows that

$$\vec{J} = -D \nabla C \quad (2)$$

where ∇ is the gradient operator and D is the constant of proportionality or diffusion coefficient. D is measured in $\text{gm sec}^{-1} \text{cm}^2 = \text{gm cm}^{-3} \text{sec}^{-1}$ and C is measured in gm cm^{-3} . It is worth noting that other investigators such as Green and Lane (1964), Bradley, Evans and Whitlaw-Gray (1946) and Romig and Long (1969) express the vapor concentration, C , in units of mol cm^{-3} rather than in units of gm cm^{-3} as is done in this work. Our C is actually the density of the gas.

Now,

$$\begin{aligned} \vec{J} \cdot \hat{n} &= -D \nabla C \cdot \hat{n} \\ &= -D \frac{\partial C}{\partial r} \end{aligned}$$

and since, for a spherical surface, \hat{n} is the unit outward normal vector in the direction of the radius, it follows that

$$\frac{\partial C}{\partial n} = \frac{\partial C}{\partial r}$$

Hence, combining equations (1) and (2) yields

$$F = -Dr^2 \frac{\partial C}{\partial r} \int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{\phi=0}^{\phi=2\pi} \sin\theta \, d\theta \, d\phi$$

which, upon carrying out the integration, gives

$$F = -4\pi Dr^2 \frac{\partial C}{\partial r} \quad (3)$$

This is the equation used by Fuchs (1959) in beginning his work on droplet evaporation.

It is now assumed that the process of evaporation is stationary, that is, the mass flux of vapor molecules across an element of the droplet surface is constant. This assumption is valid providing the concentration of vapor molecules in the far surrounding medium is much less than the concentration of molecules in the droplet. This is certainly true for droplets whose radius is larger than 1 micrometer. The assumption also requires there be no significant thermal gradients in the system. Thus, equation (3) can be written as

$$\frac{F}{r^2} dr = -4\pi D dC$$

which, upon integration, yields

$$-\frac{F}{r} = 4\pi DC + \bar{C}$$

where \bar{C} is a constant of integration. For very large values of r it is assumed that the vapor concentration is the ambient vapor concentration in the surrounding medium, C_∞ . Thus, for

$$r \rightarrow \infty \text{ it is the case that } C \rightarrow C_\infty$$

whence,

$$\bar{C} = 4\pi DC_\infty$$

Therefore,

$$\frac{F}{r} = 4\pi D(C - C_\infty)$$

or

$$F = 4\pi rD(C - C_\infty) \quad (4)$$

This is Maxwell's equation and it gives the total mass flux of vapor molecules leaving the imaginary shell of vapor surrounding the droplet to diffuse into the surrounding medium. The flux is given in terms of the diffusion coefficient of the vapor and the difference in vapor concentration in the surface layer of the droplet and in the surrounding medium. By assuming that the vapor obeys the ideal gas law

$$C = \frac{Pm}{RT} \quad (5)$$

equation (4) may be written as

$$F = \frac{4\pi Dmr}{RT} (p - p_{\infty}) \quad (6)$$

which is another form of Maxwell's equation. In equation (5), p is expressed in mm of Hg, T is expressed in degrees Kelvin and m is expressed in gm mol⁻¹. R is the universal gas constant whose value is

$$R = 62,360 \text{ cm}^3/\text{deg K/mol}$$

when the pressure is expressed in mm Hg.

Let M denote the mass (gm) of the droplet. The equation governing the rate of evaporation of the droplet is then

$$\frac{dM}{dt} = F \quad (7)$$

Hence,

$$\frac{dM}{dt} = 4\pi rD (C - C_{\infty}) \quad (8)$$

or in terms of the vapor pressure, the equation is

$$\frac{dM}{dt} = \frac{4\pi Dmr}{RT} (p - p_{\infty}) \quad (9)$$

These are the two most common forms of Maxwell's and Langmuir's equations for the temporal change of evaporation of the droplet.

To obtain expressions for the temporal variation of the surface area, radius and mass of the droplet we begin by deriving the relevant differential equations. Since the droplet is assumed to be spherical, we have

$$M = \frac{4}{3} \pi r^3 \rho$$

where ρ is the density (in gm cm⁻³) of the droplet. Under the assumption that the droplet density is constant in time, it is the case that

$$\frac{dM}{dt} = 4\pi r^2 \rho \frac{dr}{dt}$$

The surface area, S , of a sphere is

$$S = 4\pi r^2$$

and thus,

$$\begin{aligned} \frac{dS}{dt} &= 8\pi r \frac{dr}{dt} \\ &= \frac{2}{\rho r} \frac{dM}{dt} \end{aligned} \quad (10)$$

By using equation (10) with equations (8) and (9) we have

$$\frac{dS}{dt} = \frac{-8\pi Dm}{\rho} (p - p_{\infty}) \quad (11)$$

or

$$\frac{dS}{dt} = \frac{-8\pi Dm}{\rho RT} (p - p_{\infty}) \quad (12)$$

By integration we obtain

$$S = \frac{-8\pi D}{\rho} (C - C_{\infty})t + \bar{C}_1$$

and

$$S = \frac{-8\pi Dm}{\rho RT} (p - p_{\infty})t + \bar{C}_2$$

To evaluate the constants of integration, \bar{C}_1 and \bar{C}_2 , we note that when $t = t_0 = 0$,

$$S = S_0 = 4\pi r_0^2$$

where r_0 is the radius of the droplet at the beginning of the evaporation process. Therefore,

$$\bar{C}_1 = 4\pi r_0^2 = \bar{C}_2$$

and the equation governing the temporal variation of the surface area is

$$S = 4\pi r_0^2 - \frac{8\pi D}{\rho} (C - C_{\infty})t \quad (13)$$

or

$$S = 4\pi r_0^2 - \frac{8\pi Dm}{\rho RT} (p - p_{\infty})t \quad (14)$$

Since the droplet is assumed to be spherical during the entire evaporation process equations (13) and (14) may also be written as

$$r^2 = r_o^2 - \frac{2D}{\rho} (C - C_\infty) t \quad (15)$$

and

$$r^2 = r_o^2 - \frac{2Dm}{\rho RT} (p - p_\infty) t \quad (16)$$

These are the equations obtained by Maxwell and Langmuir for the temporal variation of the droplet radius.

Discussion of Maxwell and Langmuir Equations

Equations (13) and (14) show that the droplet surface area decreases linearly with time. Fuchs (1959) developed an alternative form of equation (14) to account for the fact that near an evaporating surface there must be a mass flow of vapor molecules directed away from the surface of the droplet. This flow is governed by the laws of hydrodynamics. Fuchs also showed that the correction to be applied to equation (14) to account for the mass flow is on the order of 1 percent or less for particles larger than 5 micrometers. For most insecticides, droplets whose diameters are less than 5 micrometers contain insufficient toxin to affect the target insect. Consequently, when including the effect of evaporation in estimating the probability of kill of a target insect population, it is not necessary to include Fuchs' modifications of the equations of Langmuir and Maxwell. However, when estimating the environmental impact of the drift of aerial sprays, it will be necessary to account for the ultimate deposition of very small droplets. Such an accounting will require a modification of the evaporation equations of Langmuir and Maxwell.

It is also possible to alter the previous development to account for the nonsphericity of the evaporating droplet. However, it is thought that nonspherically shaped droplets result from an extremely complex interaction of such phenomena as high-speed shear flow, turbulence, the break up of the spray injection stream, variation in surface tension,

etc. Since the analysis of this interaction has not yet been worked out, the evaporation of nonspherical droplets will not be considered. Furthermore, Pruppacher and Rasmussen (1979) found that changes in droplet shape, internal circulation within the droplet, and droplet oscillation did not reflect themselves in changes in the evaporation behavior of the droplet.

It is reasonable to assume, that spray droplets sufficiently far removed from the aircraft spray nozzle become and remain spherical due to the surface tension of the liquid. Consequently, the analysis presented in this report should be applicable to such sprays.

Equations (15) and (16) permit the calculation of the time, t_e , necessary for complete evaporation of the droplet. Thus, setting $r = 0$, gives

$$t_e = \frac{\rho r_o^2}{2D (C - C_\infty)} = \frac{\rho R T r_o^2}{2D m (p - p_\infty)} \quad (17)$$

To illustrate the use of equation (17) we compare the evaporation times of water droplets of varying initial sizes under different climatological conditions. Consider the following cases:

Case	Dry bulb temperature surrounding medium (° F)	Humidity ($p_\infty/p \times 100$) (percent)
a	50	50
b	50	60
c	60	50
d	60	60

For water we use the following values for the constants appearing in equation (17):

$$\begin{aligned} \rho &= 1 \text{ gm cm}^{-3} \\ D &= 0.240 \text{ cm}^2 \text{ sec}^{-1} \\ m &= 18 \text{ gm mol}^{-1} \\ R &= 62,360 \text{ cm}^3/\text{deg K/mol} \end{aligned}$$

The calculation is readily followed with the aid of table 1:

Table 1.--Corresponding sets of values for equation (17)

Case	Dry bulb temp. (° F)	Humidity (percent)	Wet bulb temp. (° F)	Vapor saturation pressure (mm Hg)	Temp. surrounding medium (° K)	Vapor pressure surrounding medium (mm Hg)	$\frac{\rho}{2} \frac{RT}{Dm} \times 10^{-5} \frac{1}{(p-p_{\infty})}$
a	50	50	41.9	6.769	283	3.384	6.036
b	50	60	43.6	7.280	283	4.368	7.014
c	60	50	50.3	9.309	288.556	4.655	4.418
d	60	60	52.4	10.058	288.556	6.035	5.177

Now, with the aid of equation (17), the following table for evaporation times is obtained:

Table 2.--Evaporation times of varying size water droplets for different ratios of the surrounding temperature to the relative humidity

r_o (μm)	t_e (sec)			
	Case (a) (50/50)	Case (b) (50/60)	Case (c) (60/50)	Case (d) (60/60)
1000	6036	7014	5177	4418
100	60.36	70.14	51.77	44.18
10	0.5177	0.7014	0.5177	0.4418

It is interesting to examine the consistency of these results. For example, suppose the relative humidity of the surrounding medium is held constant and the temperature increased. A comparison of the evaporation times for cases (a) and (c) and also for cases (b) and (d) shows that t_e decreases. This result agrees with our intuition; that is, the higher the temperature of the surrounding medium, the greater the

evaporation rate of the droplet. Similarly, increasing the relative humidity while holding the temperature of the medium constant, results in an increase in t_e . This is seen by comparing the evaporation times corresponding to cases (a) and (b) and to cases (c) and (d). It is possible to compare other variations of the variables and each comparison leads to a result which is in accord with our intuition.

Equations (13) and (14) reveal that, for an evaporating stationary droplet, the droplet surface area decreases linearly with time. There is ample experimental evidence to substantiate such variation providing the droplets are larger than $10\mu\text{m}$ in diameter. See for example, Green and Lane. Since the diffusion coefficient varies from one fluid to another under equal environmental conditions, the slopes of the straight lines defined by equations (13) and (14) vary accordingly. Thus, in the Maxwell-Langmuir evaporation model, the evaporation rate varies directly as the diffusion coefficient of the evaporating fluid vapor.

Equations (8) and (9) may be written as

$$\frac{1}{S} \frac{dM}{dt} = \frac{D}{r} (C - C_{\infty}) \quad (8a)$$

and

$$\frac{1}{S} \frac{dM}{dt} = \frac{D_m}{rRt} (P - P_{\infty}) \quad (9a)$$

These equations show that as the radius of the droplet approaches zero, the rate of loss of droplet mass per unit surface area increases without bound. In fact, the loss rate could exceed that predicted by statistical mechanics. Hence, the Maxwell-Langmuir model of evaporation does not properly describe the evaporation of droplets whose radii become very small. The Maxwell-Langmuir model agrees with experimental results for droplets whose radii are as small as $10\mu\text{m}$, and there is not a large variation from the experimental results for droplets whose radii are $1\mu\text{m}$ or $2\mu\text{m}$.

The next section describes a modification of these equations by Fuchs to more properly account for the evaporation of very small droplets.

Fuchs' Modification of the Maxwell-Langmuir Model

The Maxwell-Langmuir model was modified by Fuchs who assumed that:

- The diffusion of vapor molecules into the surrounding medium begins at the surface of a spherical shell of

thickness Δ surrounding the droplet. Δ is a very small distance, approximately equal to the mean free path of a molecule (fig. 1).

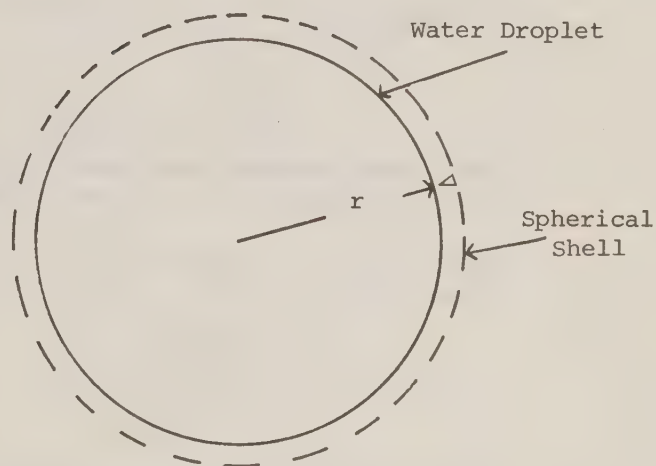


Figure 1.--Spherical shell surrounding droplet.

- The region in the shell is nearly vacuous and consequently the molecules escape from the droplet and the region exterior to the shell into the shell as if they were escaping into a vacuum.

It is known that the surface tension of a very small droplet has a significant effect on the rate of escape of molecules from the droplet. Surface tension has a similar effect on the rate of adhesion of vapor molecules in the surrounding medium which strike the droplet surface. This effect is accounted for by introducing the notion of an evaporation constant, α , whose magnitude varies with the size of the droplet. Now let C_d be the saturation vapor concentration, C_s be the vapor concentration at the surface of the spherical shell and ξ be one-fourth of the mean absolute velocity of the vapor molecules. Then, from statistical mechanics, the mass rate of accumulation of molecules from the droplet into the shell, F_d , is

$$F_d = 4\pi r^2 \alpha \xi C_d \quad (18)$$

Let F_s denote the mass rate of condensation of vapor molecules originating exterior to the shell that strike the droplet, adhere, and

become part of the droplet mass. Then

$$F_s = 4\pi r^2 \alpha \xi C_s \quad (19)$$

In these expressions ξ is given by

$$\xi = \left[\frac{kT}{\rho \pi m} \right]^{1/2} \quad (20)$$

where m is the mass of a vapor molecule and k is the gas constant per molecule.

Thus, the net or observed rate of evaporation, F_n , is

$$\begin{aligned} F_n &= F_d - F_s \\ &= 4\pi r^2 \alpha \xi (C_d - C_s) \end{aligned} \quad (21)$$

Now, with the aid of equation (4), the mass flux of droplets, F_e , escaping by diffusion into the medium surrounding the shell is

$$F_e = 4\pi(r + \Delta)D(C_s - C_\infty) \quad (22)$$

where C_∞ is the vapor concentration in the surrounding medium. A mass balance at the shell surface gives

$$F_n = F_e = F_d - F_s$$

or

$$4\pi(r + \Delta)D(C_s - C_\infty) = 4\pi r^2 \alpha \xi (C_d - C_s) \quad (23)$$

Hence, the concentration at the shell surface is

$$C_s = \frac{\alpha \xi r^2 C_d + (r + \Delta) D C_\infty}{\alpha \xi r^2 + (r + \Delta) D} \quad (24)$$

The fundamental evaporation equation is obtained by equating the time rate of change of the droplet mass with the rate at which vapor mass is escaping into the surrounding medium. Thus,

$$\frac{-dM}{dt} = F_e = 4\pi(r + \Delta)D(C_s - C_\infty)$$

or

$$\frac{-dM}{dt} = \frac{4\pi(r + \Delta)D\alpha \xi r^2 (C_d - C_\infty)}{\alpha \xi r^2 + (r + \Delta)D} \quad (25)$$

From the work of Bradley et al an estimate of Δ is

$$\Delta = \lambda \left[(m_1 + m_2)/m_1 \right]^{1/2} \quad (26)$$

where m_1 is the mean mass of a gas molecule in the surrounding medium, m_2 is the mass of an evaporating molecule and λ is the mean free path of an evaporating molecule.

Discussion of Fuchs Model

The mass rate of evaporation per unit area is

$$-\frac{1}{S} \frac{dM}{dt} = \frac{\alpha \xi D(r + \Delta)(C_d - C_\infty)}{\alpha \xi r^2 + (r + \Delta)D} \quad (27)$$

As the droplet radius approaches zero, the mass rate of evaporation per unit area is seen to approach

$$-\frac{1}{S} \frac{dM}{dt} = \alpha \xi (C_d - C_\infty) \quad (28)$$

which is a finite value. This is in contrast to the infinite value approached by the mass rate of evaporation per unit area given by the Maxwell-Langmuir model of droplet evaporation.

Expressing the mass of a spherical droplet in terms of its radius, and integrating equation (25) gives an equation for the temporal variation of the droplet radius. This equation permits the calculation of the evaporation times of droplets of various initial radii. Bradley et al. show that for small droplets of an organic substance, the Maxwell-Langmuir model predicts an evaporation time for 10 μ m droplets that is 95.6 percent of the evaporation time predicted by the Fuchs model. This is not a significant difference. In contrast, for droplets whose initial radius is 0.1 μ m, the Maxwell-Langmuir model predicts an evaporation time that is approximately 16.6 percent of that predicted by the Fuchs model whose results are in close agreement with experiment. Thus, the Fuchs model is preferred when dealing with droplets of very small radii.

Obtaining the constants appearing in the Fuchs model is more difficult than obtaining the constants used in the Maxwell-Langmuir model. Also, aerial spray droplets whose radii are less than 5 μ m are not of sufficient interest to be of concern in estimating the probability of kill of a target insect or pest. However, such droplets are certainly of interest in estimating the environmental impact of an aerial spray operation. Finally, the analytical expressions resulting from the Maxwell-Langmuir model are more readily integrated into other analyses. Consequently, we will not pursue the development of the Fuchs model.

EVAPORATION OF MOVING DROPLETS

This section of the report will describe modifications of the expression for the time rate of change of droplet radius to account for the motion of the evaporating droplet relative to the surrounding medium. The presentation will consist of a summary of the work of other investigators which is deemed most appropriate for aerial spray applications.

The technique most frequently used to account for the motion of an evaporating droplet relative to the surrounding medium is to modify the expression for the mass flux of the evaporating molecules escaping from a droplet assumed to be at rest with respect to the surrounding medium by a multiplicative term of the form

$$W = 1 + f(r, v) \quad (29)$$

where v is the velocity of the droplet relative to the surrounding medium. The function, $f(r, v)$, has the property of vanishing when the relative velocity is zero, that is, when the droplet is at rest with respect to the air. Thus, the resulting expression describing the evaporation from a moving droplet would reduce, in the case of zero relative velocity, to the expression describing the mass flux of vapor from a droplet at rest. W is frequently termed the wiping or ventilation factor.

Let the "at rest" evaporation rate be denoted by $\left. \frac{dM}{dt} \right|_0$, then the modified equation for the evaporation of a droplet is

$$\frac{dM}{dt} = [1 + f(r, v)] \left. \frac{dM}{dt} \right|_0. \quad (30)$$

With the aid of equations (8) and (9) and the expression for the time rate of change of a spherical mass, the time rate of change of the droplet radius is given by

$$\frac{dr}{dt} = \frac{D}{r\rho} [1 + f(r, v)] (C - C_\infty) \quad (31)$$

or

$$\frac{dr}{dt} = \frac{Dm}{\rho RT} [1 + f(r, v)] (P - P_\infty) \quad (32)$$

A frequently used ventilation factor is that developed by Frössling (1938). It is

$$W = 1 + a (Re)^{\frac{1}{2}} \quad (33)$$

where:

a is a constant characteristic of the evaporating substance

$$Re = \frac{2vr}{\nu} = \text{Reynold's Number}$$

$$\nu = \frac{\mu}{\rho_s} = \text{kinematic viscosity of the surrounding medium}$$

μ = viscosity of the surrounding medium, and

ρ_s = density of the surrounding medium.

It is evident that the wiping factor increases with the relative droplet velocity and the radius. A discussion of the effects of Frössling's expression on the evaporation rate of droplets is given in Green and Lane. They show that W increases by about 30 percent when the droplet radius is changed from 5 μ m to 50 μ m. There is considerable discussion in the literature concerning the numerical value of a . Most of the quoted values are such that,

$$0.20 < a < 0.35$$

Kinzer and Gunn (1951) indicate that for droplets whose radii are about $160\mu\text{m}$, the value of a is about twice that used by Frössling, while for droplets whose radii are less than $80\mu\text{m}$, the value of a should be near zero. Several investigators, such as Lambert and Quince (1969) and Green and Lane, set

$$a = 0.276 / (D_p / \mu)^{1/3} \quad (34)$$

Another form of the Frössling factor is

$$W = 1 + 0.30 (Sc)^{1/3} (Re)^{1/2} \quad (35)$$

where

$$Sc = \frac{\nu}{D} = \text{Schmidt Number}$$

This form is used by Sjenitzer (1963) and others as a starting point for deriving more complex forms of the wiping factor. Pruppacher and Rasmussen (1979) present an extensive discussion of the determination of the wiping factor for large water droplets falling at terminal velocity in air.

Still another technique used to account for the ventilation of the droplet by the relative motion of the surrounding medium consists of altering the diffusion constant in accord with the variation of the droplet temperature. For example, Penski (1978) uses the wiping factor expressed in the form given above by Sjenitzer, but assumes that the diffusion coefficient is a linear function of the droplet temperature. Such a technique requires the simultaneous calculation of the droplet temperature. Several expressions for the variation in droplet temperature have been presented in the literature. A typical expression is given by Green and Lane who further state that, for droplets whose radii are less than $100\mu\text{m}$, the heat loss by radiation can be neglected. They also conclude that, for aerosols in normal atmospheric temperature, the heat loss from the droplet by convection is so small that it can be neglected.

Since the saturation vapor pressure is usually a slowly varying function of the droplet temperature, it will be

assumed that the effects of any change in droplet temperature on the saturation vapor pressure can be ignored.

None of the above mentioned investigations distinguish between a constant or a variable relative velocity. Consequently, it is assumed that the results may be used in conjunction with the equations of motion of a falling droplet to enable the calculation of the temporal change in mass of a falling droplet.

DISCUSSION

Determining the evaporation rates of droplets comprising an aerial spray cloud is a complex task. The carrier liquid for a pesticide spray is usually oil or water based and the active toxic ingredient may be liquid or particulate. Thus, to properly determine the droplet evaporation rate, the rates of evaporation of droplets composed of multiphase liquids and particulate matter should be analyzed.

The spatial distribution of adjacent droplets also affects the evaporation rate of a single droplet within a spray cloud. In this report, for ease of analysis, it has been assumed that the evaporation rate of a single droplet can be estimated by ignoring the influence of surrounding droplets. In aerial spray applications, this assumption is probably reasonable for spray droplets far removed from the nozzle. The inclusion of the other factors in the analysis of droplet evaporation is very difficult and most of the existing work attempting to include these factors results in the construction of complex and lengthy computer codes to carry out the analysis.

Other factors affecting the evaporation rate are:

- Droplet temperature and its effect on the density of the droplet.
- Heat of vaporization, the vapor pressure and the surrounding microclimate.
- Changes in temperature, atmospheric conditions, and droplet velocity.
- Turbulence, high-speed shear flow, and fluid viscosity.

In addition, the actual field working conditions under which spraying is conducted are such that spray composition, atmospheric conditions, etc., may change considerably from one spray cycle to the next.

Finally, other phenomena, such as spray release height and swath width, are thought to be at least as important, if not more important, than droplet evaporation rate in determining the temporal and spatial variation of a spray cloud. Thus, it is reasonable to assume that a pure liquid evaporation calculation may be used as a "first approximation" in accounting for droplet evaporation in an aerial spray behavior model.

CONCLUSIONS

Both the Maxwell-Langmuir model and the Fuchs model of the evaporation of a pure liquid droplet have been discussed. Expressions have been derived which will enable the estimation of evaporation rates which may be used for comparison with experimentally obtained evaporation data. The effect of the motion of the droplet relative to the surrounding medium has been accounted for by introducing a "wiping factor." Several forms of this factor have been presented

and the relative merits of each discussed.

Very small droplets (droplets whose diameters are less than 10 micrometers) are thought to be ineffective in controlling most forest pests. Consequently, in accounting for the effect on insect mortality of the evaporation of forest insect sprays, it is appropriate to use the Maxwell-Langmuir evaporation rate equation. In contrast, the environmental problems associated with the drift of aerial sprays are known to be caused by very small droplets. Thus, when accounting for the effect of droplet evaporation on drift, Fuchs' evaporation equation should be used.

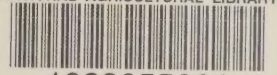
Herbicide spraying is usually characterized by droplets whose diameters are larger than 500 micrometers. For such sprays, the proper accounting of the effect of evaporation will necessitate the use of a ventilation factor.

Several experimental programs have as their objective the attainment of evaporation rate data. The use of such data in predicting the behavior of an aerial spray will require the use of interpolation. The evaporation equations derived above should provide the basis for an effective interpolation scheme.

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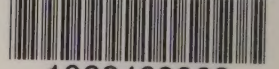
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